Determining the gravitational acceleration with a reversible pendulum

Objects of the experiments

- Measuring the oscillation periods T_1 and T_2 of a reversible pendulum for two suspension points.
- Tuning the reversible pendulum to the same oscillation period.
- Determining the gravitational acceleration from the oscillation period and the reduced length of pendulum.

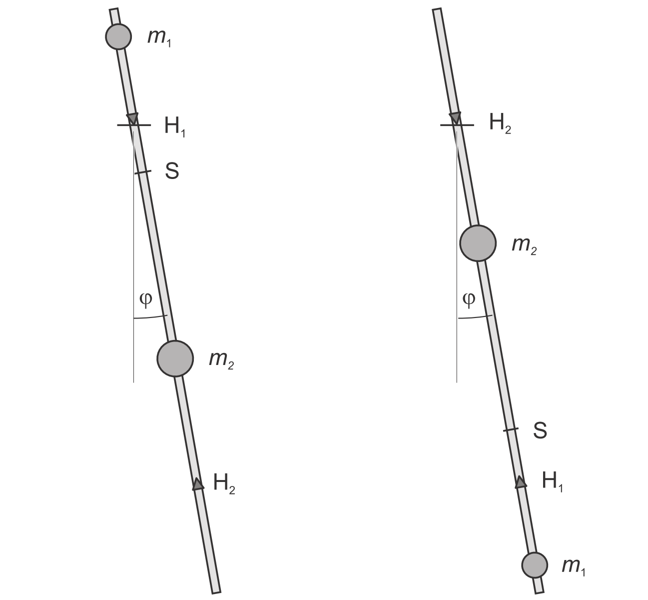

Fig. 1 Oscillations of a reversible pendulum around the suspension points H_1 and H_2.

Principles

Compound pendulum:

If a compound pendulum oscillates around its rest position with small deflections φ, the equation of motion is:

$$J \cdot \ddot{\varphi} + m \cdot s \cdot g \cdot \varphi = 0 \qquad \text{(I)}$$

J: moment of inertia around the axis of oscillation,
s: distance between the axis of oscillation and the centre of mass, g: gravitational acceleration, m: mass of the pendulum

The reduced length of the compound pendulum is defined as the quantity

$$s_r = \frac{J}{m \cdot s} \qquad \text{(II)}$$

because its oscillation period

$$T = 2\pi\sqrt{\frac{s_r}{g}} \qquad \text{(III)}$$

corresponds to that of a simple pendulum with the length s_r.

The moment of inertia J of the compound pendulum is, according to the parallel axis theorem,

$$J = J_S + m \cdot s^2 \qquad \text{(IV).}$$

J_S: moment of inertia around the centre of mass axis

Therefore the reduced length of pendulum is

$$s_r = \frac{J_S}{m \cdot s} + s \qquad \text{(V).}$$

Reversible pendulum:

The reversible pendulum is a particular type of the compound pendulum. There are two edges H_1 and H_2 that allow to choose the suspension point. Two masses m_1 = 1000 g and m_2 = 1400 g on the straight line H_1H_2 can be shifted so that the oscillation period is tunable. The goal of the tuning is to achieve equal oscillation periods around both edges. In this case, the reduced length of pendulum is equal to the distance d = 99.4 cm between the edges. This latter statement can be understood from the following consideration:

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Apparatus

1 reversible pendulum	346 111
1 steel tape measure, 2 m	311 77
1 stopclock I, 30 s/15 min	313 07

Because of Eq. (II) the pendulum oscillates around both edges H_1 and H_2 at the same oscillation period $T_1 = T_2$ if the reduced lengths of pendulum $s_{r,1}$ and $s_{r,2}$ agree. In this case we have

$$\frac{J_S}{m \cdot s_1} + s_1 = \frac{J_S}{m \cdot s_2} + s_2 \qquad \text{(VI).}$$

The relation between the distances s_1 und s_2 between the two edges and the centre of mass is given by

$$s_1 + s_2 = d \qquad \text{(VII)}$$

because the centre of mass S is located on the connecting line H_1H_2 for reasons of symmetry.

From Eqs. (VI) and (VII) a quadratic equation is obtained for determining s_1. Its solution is

$$s_1 = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} - \frac{J_S}{m}} \qquad \text{(VIII).}$$

If s_1 is inserted in Eq. (V) instead of s, the reduced length of pendulum of the tuned reversible pendulum is obtained:

$$s_r = d \qquad \text{(IX).}$$

Determining the gravitational acceleration:

For the tuned reversible pendulum

$$T^2 = 4 \cdot \pi^2 \cdot \frac{d}{g} \qquad \text{(X)}$$

follows from Eqs. (III) and (IX). The distance d between the edges is known to a high precision. After the oscillation period T of the tuned pendulum has been determined, the gravitational acceleration g is obtained from Eq. (X).

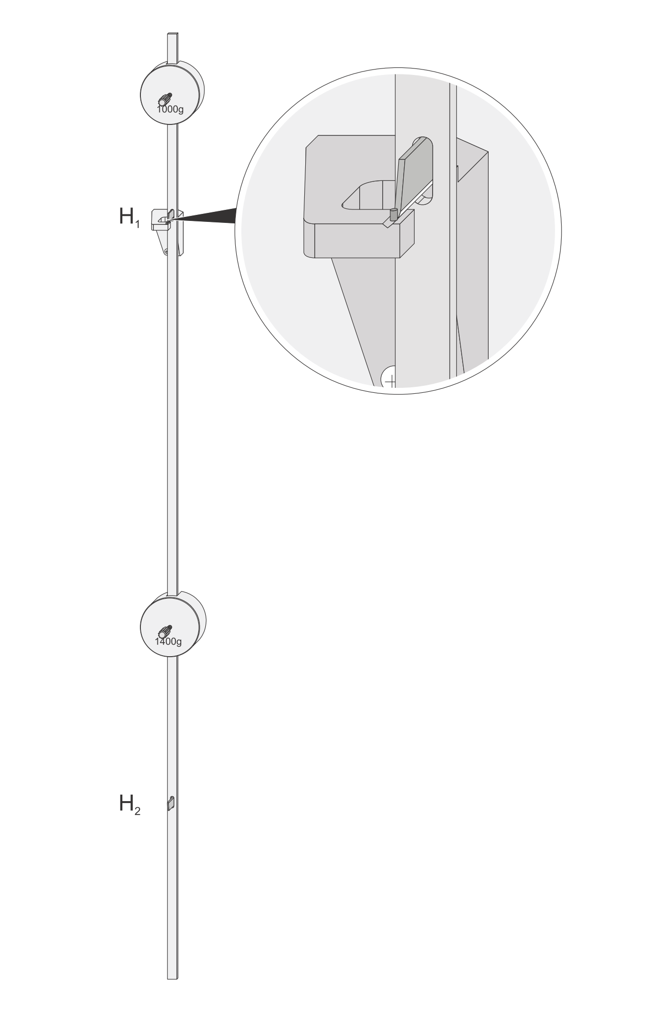
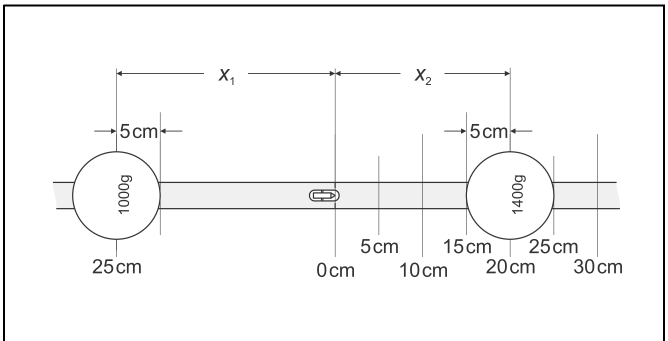


Fig. 2 Experimental setup for determining the gravitational acceleration with a reversible pendulum.

Fig. 3 Distance marks x_1 and x_2 on the pendulum rod.



Setup

The experimental setup is illustrated in Fig. 2.

– If necessary, assemble the reversible pendulum according to the instruction sheet, and mount the wall holder on a stable, non-vibrating wall.
– Make marks on the pendulum rod at x_1 = 25 cm and x_2 = 10 cm, 15 cm, 20 cm etc. (starting from the edge H_1, take into account the radius r = 5 cm of the masses, see Fig. 3)
– Fix the mass m_1 at the position x_1 = 25 cm.

Carrying out the experiment

– Carefully hang the reversible pendulum on the edge bearing of the wall holder with the edge H_1 and fix the mass m_2 at the distance x_2 = 50 cm.
– Cautiously deflect the lower end of the reversible pendulum parallel to the wall, and make it oscillate, if possible, without any vibrations.
– Measure the length of 50 oscillations and take it down as measured value $50 \cdot T_1$.

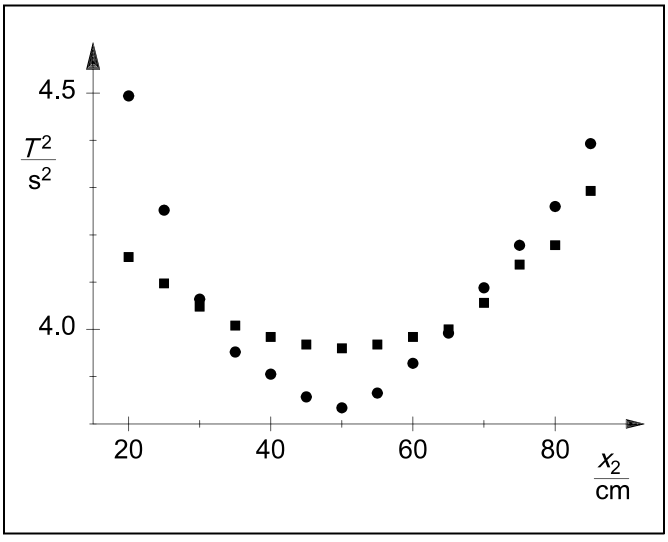
– Hang the reversible pendulum on the edge bearing with the edge H_2, and measure the period $50 \cdot T_2$.
– Slide the mass m_2 to the position x_2 = 55 cm, and measure $50 \cdot T_2$ at first and then $50 \cdot T_1$.
– Slide the mass m_2 towards the edge H_2 in steps of 5 cm; each time measure the two oscillation periods. Plot T_1^2 and T_2^2 as functions of x_2 and, if necessary, repeat the measurement of the oscillation periods.
– Next slide the mass m_2 towards the edge H_1 in steps of 5 cm starting from x_2 = 45 cm, and measure the two oscillation periods each time.

Measuring example

Table 1: Oscillation periods T_1 and T_2 around the edges H_1 and H_2, respectively, as functions of the distance x_2 between the mass m_2 and the edge H_1.

$\frac{x_2}{\text{cm}}$	$\frac{50 \cdot T_1}{\text{s}}$	$\frac{T_1^2}{\text{s}^2}$	$\frac{50 \cdot T_2}{\text{s}}$	$\frac{T_2^2}{\text{s}^2}$
20	106.0	4.494	101.9	4.153
25	103.1	4.252	101.2	4.097
30	100.8	4.064	100.6	4.048
35	99.4	3.952	100.1	4.008
40	98.8	3.905	99.8	3.984
45	98.2	3.857	99.6	3.968
50	97.9	3.834	99.5	3.960
55	98.3	3.865	99.6	3.968
60	99.1	3.928	99.8	3.984
65	99.9	3.992	100.0	4.000
70	101.1	4.088	100.7	4.056
75	102.2	4.178	101.7	4.137
80	103.2	4.260	102.2	4.178
85	104.8	4.393	103.6	4.293

Fig. 4 Squared oscillation periods around the edges H_1 (●) and H_2 (■) as functions of the distance x_2 between the mass m_2 and the edge H_1.



Evaluation and results

The two measured curves $T_1^2(x_2)$ and $T_2^2(x_2)$ intersect near x_2 = 30 cm and x_2 = 65 cm (see Fig. 4). The enlarged sections in Figs. 5 and 6 shown that the curves intersect at T^2 = 4.039 s^2 and T^2 = 4.014 s^2, respectively. With the mean value T^2 = 4.027 s^2 Eq. (X) gives

$$g = \frac{4 \cdot \pi^2 \cdot d}{T^2} = 9.74 \frac{\text{m}}{\text{s}^2}$$

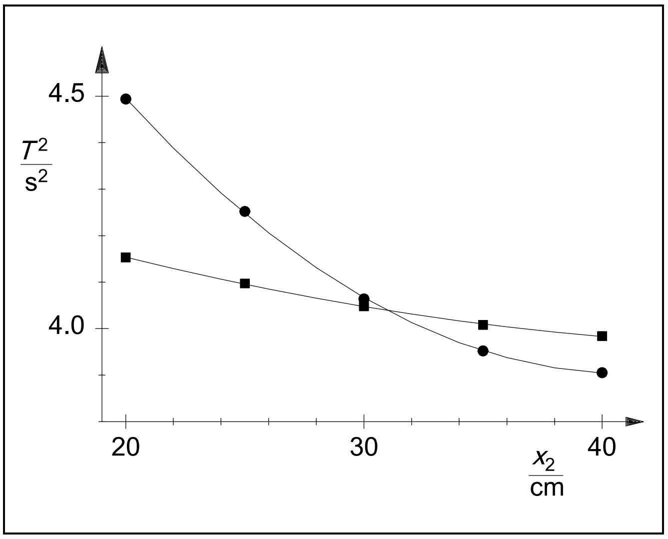


Fig. 5 Enlarged section of Fig. 4 around x_2 = 30 cm with a non-linear interpolation of the measured values. Point of intersection: (x_2 = 31 cm, T^2 = 4.039 s^2).

Fig. 6 Enlarged section of Fig. 4 around x_2 = 65 cm with a non-linear interpolation of the measured values. Point of intersection: (x_2 = 66 cm, T^2 = 4.014 s^2).